

Variational Approach to the Calculation of g_A

Benjamin J. Owen^{a,*}, Jack Dragos^a, Waseem Kamleh^a, Derek B. Leinweber^a, M. Selim Mahbub^a, Benjamin J. Menadue^a,
James M. Zanotti^a

^a*Special Research Centre for the Subatomic Structure of Matter,
School of Chemistry & Physics, University of Adelaide, South Australia, 5005, Australia*

Abstract

A long standing problem in Lattice QCD has been the discrepancy between the experimental and calculated values for the axial charge of the nucleon, $g_A \equiv G_A(Q^2 = 0)$. Though finite volume effects have been shown to be large, it has also been suggested that excited state effects may also play a significant role in suppressing the value of g_A . In this work, we apply a variational method to generate operators that couple predominantly to the ground state, thus systematically removing excited state contamination from the extraction of g_A . The utility and success of this approach is manifest in the early onset of a clear plateau in the correlation function ratio proportional to g_A . Through a comparison with results obtained via traditional methods, we show how excited state effects can suppress g_A by as much as 12% if sources are not properly tuned.

Keywords: Lattice QCD, Nucleon axial charge, Variational method

1. Introduction

In recent years, lattice calculations have taken a tremendous step towards simulating QCD at the physical point. Algorithmic and technological developments have allowed simulations to probe at or near physical quark masses on increasingly larger volumes, with finer lattice spacings and vastly increased statistics. Calculations of the ground state spectrum have yielded results consistent to within a few percent of their physical values with well controlled systematic errors [1, 2]. Naturally the next step has been to strive for this level of precision for the matrix elements of these states. Despite the remarkable consistency between lattice and experimental data for the pion form factor $F_\pi(Q^2)$, a complete description of other hadronic states, particularly the nucleon, has proven to be remarkably challenging [3, 4].

The most notable shortfall is for the nucleon axial charge, $g_A \equiv G_A(Q^2 = 0)$. In principle g_A should be relatively simple to calculate. Being an iso-vector quantity, disconnected loop contributions are absent and as we have direct access to $G_A(0)$, we circumvent the need for extrapolations in Q^2 . Unfortunately, the lattice values for g_A to date have been consistently lower than the experimental value by as much as 10–15% [5]. In an effort to account for these discrepancies, several studies have carefully examined the systematic errors present in the calculation [6–16]. In this letter we will focus on the role of excited state effects.

Recently there has been an increased effort to understand and reduce the impact of excited states on form factor calculations.

In computing these quantities, it is well understood that to ensure excited state contributions to the correlation function are sufficiently suppressed, one needs large Euclidean time separations between operators. To choose a suitable time separation one should identify the time slices where the correlation functions take on their asymptotic form. For the two commonly used sequential source techniques, this is a relatively simple procedure for the fixed current method. One simply chooses a current insertion time, t_C , once the asymptotic behaviour is observed in the two-point correlator. Results are extracted from the data once the asymptotic behaviour is observed in the three-point correlator.

For the fixed sink method, one requires knowledge of the asymptotic behaviour of the three-point correlator *a priori*. Unfortunately, the temptation to use earlier sink times in order to obtain more precise results is inescapable. These results can suffer from excited state contaminations, even if a plateau is observed with t_C . In ref. [14], it was found that for certain matrix elements, eg. $\langle x \rangle$, the source-sink separations often used in the literature were not sufficiently large to suppress excited state effects.. Nonetheless, as we move ever closer to the physical point we are naturally forced to choose earlier sink times as signal degrades much quicker.

To counter this issue, new techniques are being devised to try and control the sub-leading terms to the three point correlator. The use of the summation method [16, 17] has shown improvement upon the conventional approach, but the underlying excited states contributions are still present. It is not hard to imagine situations where these still impact significantly and alter the final result.

In this paper we take a somewhat different approach. Rather than reduce the impact of excited states through Euclidean time evolution, we seek to separate them out from the ground state

*Corresponding author

Email address: benjamin.owen@adelaide.edu.au
(Benjamin J. Owen)

at the source and sink. Drawing upon the techniques developed for excited state spectroscopy calculations, we will use the variational approach to construct interpolating fields that couple with individual energy eigenstates and use these to isolate the desired matrix elements [18, 19]. An analogous approach has been presented in [20, 21] for the study of $B^* \rightarrow B\pi$ transitions and proved to be remarkably successful. Here we apply it to g_A .

This letter is organized as follows. In Section 2 we will examine the variational method in the context of excited state spectroscopy and then outline how this method can be applied to the calculation of hadronic matrix elements. Section 3 outlines the details of this calculation. In Section 4 we present our results and compare our variational method with the traditional, single-operator approach to the calculation of g_A . Finally we provide our concluding remarks in Section 5.

2. Variational Method for Matrix Elements

The ‘variational method’ [22, 23] is a well established approach for determining the excited state hadron spectrum. It is based on the creation of a matrix of correlation functions in which different superpositions of excited state contributions are linearly combined to isolate the energy eigenstates. A diversity of excited state superpositions is central to the success of this method.

Starting from a basis of operators $\{\chi_i(x) | i = 1, \dots, N\}$, we construct a correlation matrix of two-point correlation functions. Due to the discrete nature of the lattice, we can decompose these correlation functions into a discrete sum over energy eigenstates,

$$G_{ij}(\vec{p}, t; \Gamma) = \sum_{\alpha} e^{-E_{\alpha}(\vec{p})t} Z_i^{\alpha}(\vec{p}) \bar{Z}_j^{\alpha}(\vec{p}) \text{tr} \left(\frac{\Gamma(\vec{p} + m_{\alpha})}{2E_{\alpha}(\vec{p})} \right), \quad (1)$$

where the parameters $Z_i^{\alpha}(\vec{p})$ are the coupling strengths of the interpolators $\chi_i(x)$ with the energy eigenstate of mass m_{α} and Γ projects out the desired parity. We choose new operators $\phi^{\alpha}(x)$ to be linear combinations

$$\phi^{\alpha}(x) = \sum_i v_i^{\alpha} \chi_i(x), \quad \bar{\phi}^{\alpha}(x) = \sum_j u_j^{\alpha} \bar{\chi}_j(x), \quad (2)$$

with a suitable choice of coefficients v_i^{α} and u_j^{α} , such that these interpolators couple to a single energy eigenstate,

$$\langle \Omega | \phi^{\beta}(0) | \alpha, p, s \rangle = \delta^{\alpha\beta} \mathcal{Z}_{\alpha}(\vec{p}) \sqrt{\frac{m_{\alpha}}{E_{\alpha}(\vec{p})}} u(p, s). \quad (3)$$

From Eqs. (1) and (3) we find that the necessary values for v_i^{α} and u_j^{α} are the solutions of the following eigenvalue equations

$$v_i^{\alpha}(\vec{p}) [G(\vec{p}, t_0 + \Delta t) (G(\vec{p}, t_0))^{-1}]_{ij} = c^{\alpha} v_j^{\alpha}(\vec{p}), \quad (4)$$

$$[(G(\vec{p}, t_0))^{-1} G(\vec{p}, t_0 + \Delta t)]_{ij} u_j^{\alpha}(\vec{p}) = c^{\alpha} u_i^{\alpha}(\vec{p}), \quad (5)$$

where the eigenvalue $c^{\alpha} = e^{-m_{\alpha}\Delta t}$.

It is important to note that both (4) and (5) are evaluated for a given momentum \vec{p} and so the diagonalisation condition is

only satisfied when we project with the relevant coefficients as follows:

$$v_i^{\alpha}(\vec{p}) G_{ij}(\vec{p}, t; \Gamma) u_j^{\beta}(\vec{p}) \propto \delta^{\alpha\beta}. \quad (6)$$

Thus the two-point correlation function for the state $|\alpha, p\rangle$ is

$$G^{\alpha}(\vec{p}, t; \Gamma) \equiv v_i^{\alpha}(\vec{p}) G_{ij}(\vec{p}, t; \Gamma) u_j^{\alpha}(\vec{p}). \quad (7)$$

We can extract the mass m_{α} from $G^{\alpha}(\vec{p} = 0, t)$ in the standard way.

To understand how we can utilise the variational method for use in form factor calculations, we must firstly identify the terms present in the three-point correlation function,

$$G_{ij}^{\mu}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = \sum_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}' \cdot \vec{x}_2} e^{+i(\vec{p}' - \vec{p}) \cdot \vec{x}_1} \text{tr} \left(\Gamma' \langle \Omega | \chi_i(x_2) J^{\mu}(x_1) \bar{\chi}_j(0) | \Omega \rangle \right). \quad (8)$$

Sandwiching the current operator between two complete sets of states we end up with three terms, the vertex amplitude, $\langle \beta, p', s' | J^{\mu}(0) | \alpha, p, s \rangle$, and the coupling terms $\langle \Omega | \chi_i(0) | \beta, p', s' \rangle$ and $\langle \alpha, p, s | \bar{\chi}_j(0) | \Omega \rangle$,

$$G_{ij}^{\mu}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = \sum_{\alpha, \beta} e^{-E_{\beta}(\vec{p}') (t_2 - t_1)} e^{-E_{\alpha}(\vec{p}) t_1} Z_i^{\beta}(\vec{p}') \bar{Z}_j^{\alpha}(\vec{p}) \sqrt{\frac{m_{\alpha} m_{\beta}}{E_{\alpha}(\vec{p}) E_{\beta}(\vec{p}')}} \text{tr} \left(\Gamma' \sum_{s', s} u(p', s') \langle \beta, p', s' | J^{\mu}(0) | \alpha, p, s \rangle \bar{u}(p, s) \right). \quad (9)$$

The coupling parameters take the same form as they did in the calculation of the two-point correlator with two key differences. The inclusion of a current means that the initial and final momenta need not be the same. Furthermore, there also exists the possibility that the initial and final energy eigenstates are not the same. That is, the current can induce a transition between states. For this calculation the necessary expression is

$$G^{\alpha\mu}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') = v_i^{\alpha}(\vec{p}') G_{ij}^{\mu}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') u_j^{\alpha}(\vec{p}). \quad (10)$$

To isolate the matrix element from the three-point function, we construct a ratio in the standard way. In this work we shall use the ratio defined in [24]. For the state α the necessary ratio is,

$$R^{\alpha\mu}(\vec{p}', \vec{p}; \Gamma', \Gamma) = \sqrt{\frac{G^{\alpha\mu}(\vec{p}', \vec{p}; t_2, t_1; \Gamma') G^{\alpha\mu}(\vec{p}, \vec{p}'; t_2, t_1; \Gamma')}{G^{\alpha}(\vec{p}, t_2; \Gamma) G^{\alpha}(\vec{p}', t_2; \Gamma)}}. \quad (11)$$

Key to this approach is the utilisation of a basis of operators in which there is diversity in the overlap with various excited states. As there are a limited number of local bilinear operators for a given J^{PC} , a great deal of work has been made by various groups in increasing the number of available operators. Here we choose to use fermion source and sink smearing as a method of extending our operator basis, as outlined in [25, 26].

3. Calculation Details

For this calculation we make use of the PACS-CS (2+1)-flavour dynamical-QCD gauge field configurations [27] made available through the ILDG [28]. These configurations are generated using a non-perturbatively $O(a)$ -improved Wilson fermion action and Iwasaki gauge action. The value $\beta = 1.90$ results in a lattice spacing $a = 0.091$ fm, determined via the static quark potential. With dimensions $32^3 \times 64$, these ensembles correspond to a spatial length of $L = 2.9$ fm. As the intention of this paper is to examine whether the variational approach is an improvement upon traditional techniques, we will consider only the light quark mass that corresponds to $m_\pi \approx 290$ MeV. The resulting value of $m_\pi L = 4.26$ is comparable to the values used by most groups.

In this work we are primarily interested in isolating the ground state and so have chosen to use a small variational basis upon which to perform our correlation matrix analysis. We use gauge-invariant Gaussian smearing in the spatial dimensions only with smearing fraction $\alpha = 0.7$ [29]. We consider four levels of smearing with the optimal choice found in [29], these being 16, 35, 100 and 200, applied to the standard, local proton interpolator

$$\chi_1(x) = \epsilon^{abc} [u^{aT}(x) C \gamma_5 d^b(x)] u^c(x),$$

thus allowing for construction of a correlation matrix of dimension up to 4×4 . In Table 1 we list the rms-radii for our choice of smearing parameters. We choose to use variational parameters $t_0 = 18$ and $\Delta t = 2$, again taken from [29], where it was found that this choice produced best balance between systematic and statistical uncertainties.

Table 1: The rms radii for the various levels of smearing considered in this work.

Sweeps of smearing	rms radius (fm)
16	0.216
35	0.319
100	0.539
200	0.778

To extract the nucleon axial charge we are interested in the matrix element $\langle p(p', s') | A_\mu^{ud} | n(p, s) \rangle$ where $A_\mu^{ud} = \bar{u} \gamma_\mu \gamma_5 d$. This vertex can be expressed via two independent form factors, the axial form factor $G_A(Q^2)$ and the induced pseudoscalar form factor $G_P(Q^2)$, as

$$\langle p(p', s') | A_\mu^{ud} | n(p, s) \rangle = \left(\frac{m^2}{E_{p'} E_p} \right)^{1/2} \bar{u}_p(p', s') \left[\gamma_\mu \gamma_5 G_A(Q^2) + \gamma_5 \frac{q_\mu}{2m} G_P(Q^2) \right] u_n(p, s), \quad (12)$$

where $q_\mu = p'_\mu - p_\mu$ and $Q^2 = -q^2$. Using isospin symmetry, one can show that the flavour-changing current

$$\langle p(p', s) | A_\mu^{ud} | n(p, s) \rangle = \langle p(p', s) | A_\mu^{u-d} | p(p, s) \rangle,$$

where $A_\mu^{u-d} = \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d$. We choose to calculate g_A using A_μ^{u-d} .

As we are interested in $G_A(Q^2 = 0)$, it suffices to consider the case where the incoming and outgoing momenta are the same, in particular we choose to work in the nucleon rest frame as this will provide the smallest statistical uncertainties. This will mean that the left and right eigenvectors required to project out the three-point function will now correspond to the same momenta.

We choose to insert our fermion source at $t_0 = 16$. For the calculation of the three-point functions we use a local axial vector current calculated using a sequential source technique with the current held fixed and inserted at $t_C = 21$, well past the onset of asymptotic behaviour for the projected two-point function. We choose to use $\mu = 3$ for the current with the corresponding projection matrix being $\Gamma' = \Gamma^3 = \Gamma_4 \gamma_5 \gamma^3$, where $\Gamma_4 \equiv \frac{1}{2}(I + \gamma_0)$. The value for the axial renormalisation constant $Z_A = 0.781$ was determined non-perturbatively in [30] using a Schrödinger functional scheme.

The resulting expression from which we extract g_A is

$$g_A^{CM} = \frac{v_i^0(0) G_{ij}^3(0, 0; t_2, t_1; \Gamma_3) u_j^0(0)}{v_i^0(0) G_{ij}(0, t; \Gamma_4) u_j^0(0)}. \quad (13)$$

As a comparison, we also evaluate g_A using a single correlator from smeared source to point sink and smeared source to smeared sink. These are indicative of results one would extract from a traditional approach. We examine the four source smearing levels used in the correlation matrix calculation.

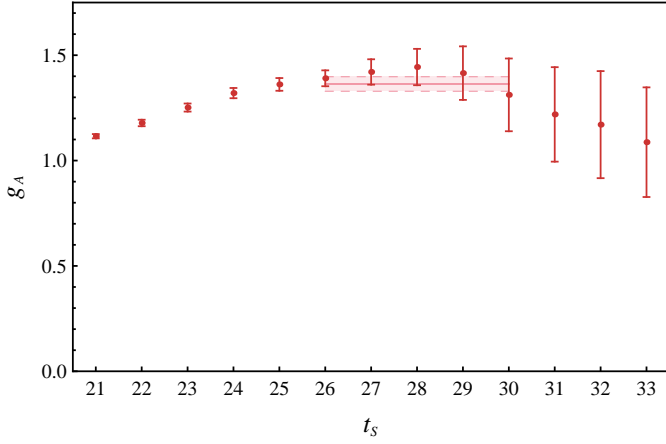
4. Results

In Fig. 1 we present the bare values of g_A with increasing sink time t_s for the smeared source to point sink, smeared source to smeared sink (both with 35 sweeps of smearing) and our variational method respectively. Between the traditional approach (upper two plots) and the variational approach (bottom plot), we can see significant differences in the overall shape of the plateau.

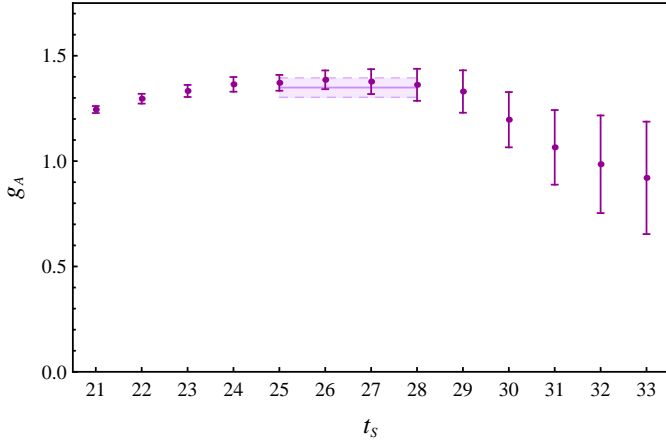
For the smeared source to point sink (upper plot) the Euclidean time suppression of excited state contributions manifests itself as a steady increase in the extracted value. This trend in the data does not have a clear endpoint and so there is no definite plateau. Guided by the χ_{dof}^2 obtained via a covariance matrix analysis, the earliest time slice one could consider is $t_S = 25$, but what is clear is that we are forced to consider fit windows uncomfortably close to regions dominated by noise.

By smearing the sink as well as the source, there is a definite improvement in the quality of the plateau. The excited state behaviour is again present as a steady increase in the value of g_A , but somewhat suppressed. In this case there is a definite plateau observed at $t_S = 24$, which is supported by the χ_{dof}^2 . Unfortunately, this is again somewhat close to the region where signal is lost to noise.

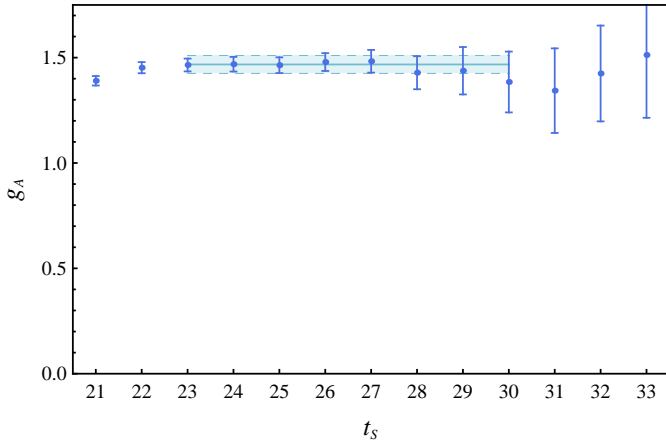
In Fig. 1 (c) we see quite a different situation. Our variational approach yields extremely clean results with rapid ground state



(a) Smeared source to point sink



(b) Smeared source to smeared sink



(c) Variational approach

Figure 1: A comparison of un-renormalized g_A as a function of sink time. The first two figures are using the traditional approach of *smeared* source \rightarrow *point* sink and, *smeared* source \rightarrow *smeared* sink, both for 35 sweeps of smearing. The final figure is the result from a 4×4 correlation matrix.

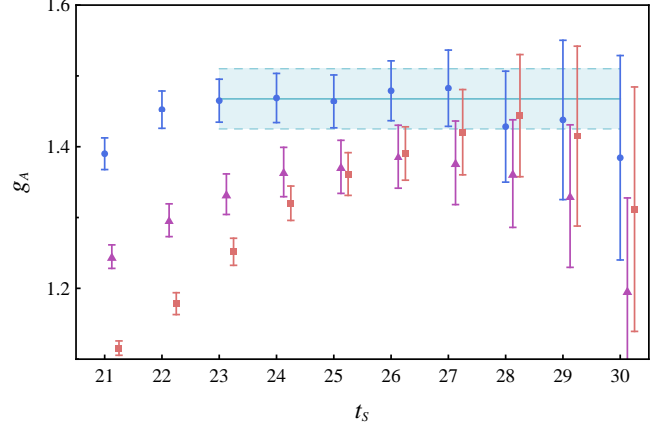


Figure 2: An overlay of the results from Fig. 1. The data sets have been offset from the time slice for clarity – the circles (blue) are the results for the variational approach, the triangles (purple) are the *smeared* source \rightarrow *smeared* sink, while the squares (red) are the *smeared* source \rightarrow *point* sink. The fitted value from the variational approach has been included (blue shaded region) to highlight where the traditional approach is consistent with the improved method.

dominance. The systematic rise in the data is no longer present and the onset of the plateau is within two time slices of the current insertion.

In Fig. 2 we have overlaid the three datasets to highlight the excited state behaviour between the traditional and variational approach. If we look carefully at the variational approach, we can see that some excited state contamination is present immediately after the current, but this is short lived. It is worth noting that this is a consequence of the limited size of our variational basis. As is highlighted in [21], an $n \times n$ correlation matrix allows one to isolate out the n lightest states in the given channel and so the sub-leading contributions will come from the $n^{\text{th}} + 1$ state. In the case of the ground state, these contributions will be short-lived due to the large mass splitting between the ground state and $n^{\text{th}} + 1$ excited state. If one were to construct a basis whose dimension was the number of states in the given channel, then it would be possible to completely isolate each state.

What is of most concern in Fig. 2 is the lack of overlap between the results of the traditional approach and those of our variational method which is free of excited state contamination. In Table 2 we list those fits, for the three data sets with the strict criterion that the χ^2_{dof} lies between 0.800 and 1.200. In both data sets employing the traditional approach, we can obtain good fits with small uncertainties if we choose to begin fitting around $t_S = 25$ or 26, but find that the results are significantly small. The correct result can be extracted from these datasets if we choose to fit at later time slices around $t_S = 28$, but the resulting values have unattractively large uncertainties, as they are close to the onset of noise. Alarming, as we move the fit window to later times, the central value increases. Between 25–30 and 28–30 we observe a systematic variation of 6% in the value g_A . It is clear that in this case, we have little control over the excited state systematics. In contrast, between the various fit windows

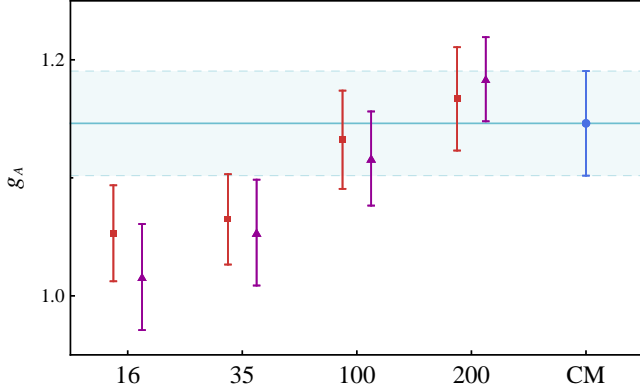


Figure 3: Comparison of the renormalized value of g_A . The first four pairs of points are the results for the traditional, point sink (squares) and smeared sink (triangles) approach with increasing levels of smearing to the right. The rightmost point (circle) is the result extracted using variational approach. There is a clear dependence on the level of smearing to the extracted result.

on the data from the variational approach we find the variation in the fits is considerably smaller than the smallest statistical uncertainty.

It is worth considering how the level of smearing affects the extracted value of g_A . In Fig. 3, we present the renormalized g_A considering each of the four smearings used to construct our variational basis. What we find is a dependence on the level of smearing used in the calculation. It appears that for low levels of smearing the extracted result can be significantly lower, with the smallest level of smearing differing by up to 15% from our improved, variational result. From this evidence, it is clear that if the smearing level is not properly tuned then excited state effects significantly impact the extracted result for g_A . Increased smearing will improve the extracted result, but there is an optimal value.

In principle, one could tune the smearing so that the optimal overlap is observed with the ground state. By using a point source propagator and tuning the smearing through the sink via the two-point correlator, outlined in [31], one removes the need for expensive inversions for each smearing. Unfortunately, there is no guarantee that the optimal level of smearing for a given quark mass, β value, momentum or operator will be universal and so one must tune the smearing for each set of parameters under consideration. Immediately, one can find appeal in the variational approach as there is no longer a need to tediously tune the operators to match the ground state.

The variational approach provides us with a systematic framework for constructing operators whereby we have not suppressed, but instead removed the contributions of the nearby states. To see how small one could make the variational basis so as to obtain the correct result, we examined all possible subsets of our variational basis. The results are displayed in Fig. 4. To ensure excited state effects are well suppressed it appears that the higher levels of smearing are key. Furthermore, clean results require at least a 3×3 correlation matrix.

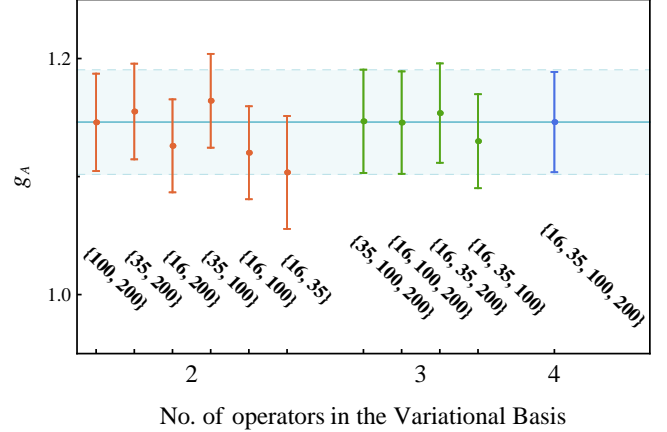


Figure 4: Results for g_A with different number and combinations of operators used in the variational analysis.

5. Conclusion

In this letter we have illustrated how the variational approach can be used to eliminate excited state effects from the calculation of the nucleon axial-vector coupling constant g_A . These effects act to suppress lattice simulation results for g_A .

The method is general and would be ideally suited to calculations of form factors where the variational approach could be applied separately for each choice of source-sink momentum combination. Another quantity that has so far proved elusive for lattice calculations and could benefit from our approach is the quark momentum fraction, $\langle x \rangle$, which is notorious for producing lattice results that are more than 50% larger than phenomenological determinations (see [32] for a review).

Future investigations will accurately calculate g_A at a variety of quark masses and connect these results to Nature via finite-volume chiral effective field theory.

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References

- [1] Z. Fodor, C. Hoelbling, Light Hadron Masses from Lattice QCD, Rev.Mod.Phys. 84 (2012) 449.
- [2] C. Hoelbling, Light hadron spectroscopy and pseudoscalar decay constants, PoS LATTICE2010 (2010) 011.
- [3] P. Hägler, Hadron structure from lattice quantum chromodynamics, Phys.Rept. 490 (2010) 49–175.
- [4] C. Alexandrou, Hadron Structure and Form Factors, PoS LATTICE2010 (2010) 001.
- [5] H. W. Lin, Lattice Hadron Structure: Applications within and beyond QCD, PoS LATTICE2012 (2012) 013.

Table 2: Un-renormalized values of g_A from fit windows which give a covariance matrix based χ^2_{dof} between 0.800 and 1.200. We note how the value of g_A increases for the traditional approach as we move the fit window to later times. In contrast, the variational approach is stable across all windows with the desired χ^2_{dof} . The datasets are identified as (a) Traditional approach with point sink, (b) Traditional approach with smeared sink and (c) Variational approach.

(a)			(b)			(c)		
Fit Window	g_A	χ^2_{dof}	Fit Window	g_A	χ^2_{dof}	Fit Window	g_A	χ^2_{dof}
25 – 27	1.36(4)	1.168	24 – 30	1.35(4)	1.161	23 – 30	1.47(5)	0.848
25 – 30	1.35(4)	1.100	24 – 31	1.34(4)	1.104	23 – 31	1.47(5)	0.818
25 – 31	1.34(4)	0.951	25 – 28	1.35(5)	0.926	24 – 29	1.48(4)	0.848
26 – 27	1.38(4)	0.808	25 – 29	1.34(5)	0.812	24 – 30	1.47(4)	0.988
26 – 30	1.36(4)	1.077	26 – 30	1.38(6)	1.100	24 – 31	1.47(4)	0.932
26 – 31	1.36(4)	0.902	26 – 31	1.38(6)	0.952	25 – 29	1.47(4)	0.951
27 – 31	1.38(5)	1.011	27 – 31	1.36(8)	1.148	25 – 30	1.47(5)	1.120
28 – 30	1.44(7)	1.129	28 – 31	1.32(8)	1.082	25 – 31	1.47(5)	1.040
29 – 31	1.49(12)	0.994				26 – 28	1.47(5)	1.091
						26 – 29	1.48(5)	1.184
						26 – 31	1.48(6)	1.146

- [6] J. Bratt, et al., Nucleon structure from mixed action calculations using 2+1 flavors of asqtad sea and domain wall valence fermions, Phys.Rev. D82 (2010) 094502.
- [7] A. Ali Khan, et al., Axial coupling constant of the nucleon for two flavours of dynamical quarks in finite and infinite volume, Phys.Rev. D74 (2006) 094508.
- [8] T. Yamazaki, et al., Nucleon axial charge in 2+1 flavor dynamical lattice QCD with domain wall fermions, Phys.Rev.Lett. 100 (2008) 171602.
- [9] R. Edwards, et al., The Nucleon axial charge in full lattice QCD, Phys.Rev.Lett. 96 (2006) 052001.
- [10] S. Sasaki, K. Orginos, S. Ohta, T. Blum, Nucleon axial charge from quenched lattice QCD with domain wall fermions, Phys.Rev. D68 (2003) 054509.
- [11] C. Alexandrou, et al., Axial Nucleon form factors from lattice QCD, Phys.Rev. D83 (2011) 045010.
- [12] D. Pleiter, et al., Nucleon form factors and structure functions from N(f)=2 Clover fermions, PoS LATTICE2010 (2010) 153.
- [13] P. Hägler, et al., Nucleon Generalized Parton Distributions from Full Lattice QCD, Phys.Rev. D77 (2008) 094502.
- [14] S. Dinter, et al., Precision Study of Excited State Effects in Nucleon Matrix Elements, Phys.Lett. B704 (2011) 89–93.
- [15] N. L. Hall, A. W. Thomas, R. D. Young, J. M. Zanotti, Volume Dependence of the Axial Charge of the Nucleon (2012).
- [16] S. Capitani, et al., The nucleon axial charge from lattice QCD with controlled errors, Phys.Rev. D86 (2012) 074502.
- [17] S. Capitani, et al., Excited state systematics in extracting nucleon electromagnetic form factors (2012).
- [18] B. Owen, W. Kamleh, D. B. Leinweber, M. S. Mahbub, B. Menadue, Correlation matrix methods for excited meson form factors in Full QCD, PoS LATTICE2012 (2012) 173.
- [19] B. J. Menadue, W. Kamleh, D. B. Leinweber, M. S. Mahbub, B. J. Owen, Electromagnetic Form Factors of the $\Lambda(1405)$ in (2+1)-flavour Lattice QCD, PoS LATTICE2012 (2012) 178.
- [20] J. Bulava, M. Donnellan, R. Sommer, On the computation of hadron-to-hadron transition matrix elements in lattice QCD, JHEP 1201 (2012) 140.
- [21] F. Bernardoni, et al., B-physics from HQET in two-flavour lattice QCD (2012).
- [22] C. Michael, Adjoint Sources in Lattice Gauge Theory, Nucl.Phys. B259 (1985) 58.
- [23] M. Lüscher, U. Wolff, How to calculate the elastic scattering matrix in two-dimensional quantum field theories by numerical simulation, Nucl.Phys. B339 (1990) 222–252.
- [24] J. N. Hedditch, et al., Pseudoscalar and vector meson form-factors from lattice QCD, Phys.Rev. D75 (2007) 094504.
- [25] M. S. Mahbub, et al., Isolating Excited States of the Nucleon in Lattice QCD, Phys.Rev. D80 (2009) 054507.
- [26] M. S. Mahbub, A. O. Cais, W. Kamleh, D. B. Leinweber, A. G. Williams, Positive-parity Excited-states of the Nucleon in Quenched Lattice QCD, Phys.Rev. D82 (2010) 094504.
- [27] S. Aoki, et al., 2+1 Flavor Lattice QCD toward the Physical Point, Phys.Rev. D79 (2009) 034503.
- [28] M. G. Beckett, et al., Building the International Lattice Data Grid, Comput.Phys.Comm. 182 (2011) 1208–1214.
- [29] M. S. Mahbub, W. Kamleh, D. B. Leinweber, P. J. Moran, A. G. Williams, Roper Resonance in 2+1 Flavor QCD, Phys.Lett. B707 (2012) 389–393.
- [30] S. Aoki, et al., Non-perturbative renormalization of quark mass in $N_f = 2 + 1$ QCD with the Schroedinger functional scheme, JHEP 1008 (2010) 101.
- [31] D. S. Roberts, W. Kamleh, D. B. Leinweber, M. S. Mahbub, B. J. Menadue, Accessing High Momentum States In Lattice QCD, Phys.Rev. D86 (2012) 074504.
- [32] D. B. Renner, Status of Average-x from Lattice QCD, AIP Conf.Proc. 1369 (2011) 29–36.